International Journal of Theoretical Physics, Vol. 34, No. 9, 1995

On The Thermodynamic Properties of the Quantum Vacuum

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Received November 2, 1994

Several thermodynamic relations for the vacuum state have been derived by assuming that it behaves like a relativistic perfect simple fluid. Unlike the usual fluids, the vacuum becomes hotter when it undergoes an adiabatic expansion (TV^{-1} = const). A new Lorentz-invariant spectrum for the vacuum is also suggested which is compatible with the usual equation of state $p = -\rho$ and the other thermodynamic constraints. Some cosmological consequences of these results have also been discussed.

Different conceptions about the quantum vacuum are found in the literature (Planck, 1911, 1912; Einstein and Stern, 1913; Casimir, 1948; Marshall, 1963; Boyer, 1969, 1980; Gliner, 1966; Zeldovich, 1968; Grøn, 1986; Weinberg, 1989; Iliopolus *et al.*, 1975; Meyers, 1987; Blau *et al.*, 1988; Saunders and Brown, 1991). The first arose already in the years of the old quantum theory (Planck, 1911, 1912). It is closely related to the possible existence of a zero-point energy for blackbody radiation. The random background radiation corresponding to the zero-point field is the key ingredient of so-called stochastic electrodynamics (SED) (Marshall, 1963; Boyer, 1969, 1980). Later, with the development of quantum field theories (QFT), a new concept arose, namely: the vacuum state is the one which has no quanta in it. Technically, this means that the effect of the annihilation operator is zero. Although much effort has been made to reconcile both approaches, they are fundamentally different on physical grounds. Particularly, in the SED framework, the random background is a real field endowed with a well-defined frequency spectrum [$\rho_T(\nu) \sim \nu^3$], while for QFT, the vacuum is filled with virtual pairs of particles fully transparent to ordinary particle detectors.

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0020-7748/95/0900-1835$07.50/0

More recently, it was remarked that Lorentz invariance of the vacuum state requires an energy-momentum tensor (EMT) of the form (Gliner, 1966; Zeldovich, 1968)[3]

$$\langle T_{\mu\nu} \rangle = \langle \rho \rangle \eta_{\mu\nu} \tag{1}$$

where ρ is the vacuum energy density and $\eta_{\mu\nu}$ is the Minkowski tensor. Thus, the EMT of the vacuum describes a particular relativistic perfect simple fluid, for which the equation of state is $p = -\rho$. This result was also explicitly obtained in the QFT domain through a relativistically invariant regularization of the vacuum energy density and the corresponding pressure (Zeldovich, 1968). On the other hand, since the EMT (1) is divergenceless, the energy density ρ is constant in space-time, leading to a situation characterized in the realm of general relativity by the cosmological constant stress. Such a property has profound physical consequences for the interface uniting particle physics and cosmology. In fact, it is very puzzling that the cosmological upper bound of the effective vacuum energy density differs from natural theoretical expectations in QFT by more than 100 orders of magnitude (Weinberg, 1989). Note also that performing a change of inertial frame, the energy density of a perfect fluid transforms (Weinberg, 1971; Landau and Lifschitz, 1975)

$$\rho' = \frac{\rho + pv^2/c^2}{1 - v^2/c^2} \tag{2}$$

where v is the relative velocity between the frames. Therefore, it follows from the equation of state $p = -\rho$ that the energy density of the vacuum is a Lorentz-invariant quantity, regardless of the form of its frequency spectrum. In other words, all inertial observers are comoving with the vacuum background. Clearly, such a statement can be extended for a general space-time by just suppressing the word inertial.

In this paper we are mainly interested in the above macroscopic point of view. In principle, if models based on microphysics have failed to illuminate the nature of the quantum vacuum, an alternative and secure way is to consider a thermodynamic approach, since its conclusions are not dependent upon the microscopic details. As we shall see, by regarding the vacuum state of any bosonic or fermionic field as an unusual substance described by $p = -\rho$, we can easily deduce its overall thermodynamic properties. As in the case of blackbody radiation, such properties shed light on the true nature of the vacuum, determining, for instance, the general form of its frequency spectrum.

The thermodynamic states of a relativistic simple fluid are characterized by an EMT $T^{\alpha\beta}$, a particle current N^{α}, and an entropy current S^{α}. For a

[3] A pedagogical approach can be found in Grøn (1986).

perfect fluid such quantities are defined by (for completeness we will consider the general relativistic framework)

$$T^{\alpha\beta} = (\rho + p)u^\alpha u^\beta - pg^{\alpha\beta} \tag{3}$$

$$N^\alpha = nu^\alpha \tag{4}$$

$$S^\alpha = n\sigma u^\alpha \tag{5}$$

where ρ is the energy density, p is the pressure, n is the number density, and σ is the specific entropy (per particle). The variables ρ, p, u, and σ are related to the temperature T by the Gibbs law (Weinberg, 1971; Landau and Lifschitz, 1975)

$$nT\,d\sigma = d\rho - \frac{\rho + p}{n}\,dn \tag{6}$$

while the basic quantities are constrained by the following relations:

$$T^{\alpha\beta}{}_{;\beta} = 0 \tag{7}$$

$$N^\alpha{}_{;\alpha} = 0 \tag{8}$$

$$S^\alpha{}_{;\alpha} = 0 \tag{9}$$

where the semicolon denotes covariant derivative. Equations (7) and (8) express, respectively, the laws of conservation of energy momentum and number of particles, whereas (9) is the thermodynamic second law restricted to an adiabatic flow ("equation of continuity" for entropy).

By considering T and n as independent thermodynamic variables, one finds from equations (3)–(9) that the temperature obeys the following evolution equation (Calvão and Lima, 1989; Calvão *et al.*, 1992):

$$\frac{\dot{T}}{T} = \left(\frac{\partial p}{\partial \rho}\right)_n \frac{\dot{n}}{n} \tag{10}$$

where an overdot means the comoving time derivative (for instance, $\dot{T} \equiv u^\alpha T_{;\alpha}$).

Now, for the sake of generality, let us consider the "gamma-law" equation of state:

$$p = (\gamma - 1)\rho \tag{11}$$

where the "adiabatic index" γ is 4/3 for photons ($p = \frac{1}{3}\rho$) and zero for the vacuum fluid ($p = -\rho$). Usually, in the cosmological context γ ranges over the interval[4] $0 \leq \gamma \leq 2$. With the choice (11), a straightforward integration of (10) furnishes

$$Tn^{1-\gamma} = \text{const} \tag{12}$$

and since n scales with V^{-1}, where V is the volume of the considered portion within the fluid, equation (12) assumes the form

$$T^{1/(\gamma-1)}V = \text{const} \tag{13}$$

which is the usual adiabatic law for fluids with conserved net number of particles (Calvão *et al.*, 1992). For photons the above expression reduces to $T^3V = \text{const}$, a well-known result, while for the vacuum state ($\gamma = 0$) we obtain

$$\frac{T}{V} = \text{const} \tag{14}$$

We have therefore reached the conclusion that the vacuum becomes hotter if it undergoes an adiabatic expansion. Such a result must be compared with those of the usual theory of fluids, for which $\gamma > 1$ ($p > 0$). As a matter of fact, the temperature of an expanding adiabatic γ-fluid with negative pressure ($\gamma < 1$) grows, as one can see from (13). In this connection we recall that thermodynamic states with negative pressure are metastable, but they are not forbidden by any law of nature. Such states appear naturally in some phase transitions, as happens in an overheated van der Waals liquid. Systems with negative pressure are also hydrodynamically unstable for bubbles and cavity formation and a spontaneous collapse could also be expected (Landau and Lifschitz, 1980). In the case of the vacuum, it is tempting to speculate whether such collapse may be responsible for matter creation "from nothing," with the particles being ultimately described as a kind of vacuum condensation.

It should be emphasized that in the derivation of (13) the conservation of the number of particles was explicitly used. However, there are at least two cases (just photons, and the vacuum) for which the meaning of such an assumption needs to be clarified. For $p = \frac{1}{3}\rho$ we see from (12) that n scales with T^3. Of course, n must be interpreted here as the average number density of photons since its chemical potential is zero. As is well known, such an interpretation is consistent with the Planck distribution, which furnishes $n = \int_0^\infty [\rho_T(\nu)/h\nu]\, d\nu = bT^3$, where b is a constant (Landau and Lifschitz, 1980).

[4]The lower and upper limits are determined from causality requirements since the sound velocity is $v_s = c\,|(\partial p/\partial\rho)_\sigma|$. The case $p = \rho$ is the Zeldovich stiff matter (Zeldovich, 1962), whereas $p = -\rho$ corresponds to the vacuum state.

In what follows we assume that similar considerations hold for the vacuum state ($\gamma = 0$), for which equation (12) yields

$$n = \frac{\text{const}}{T} \tag{15}$$

Hence, we see that for the vacuum state, the average density of particles decreases with growing T. In the limit $T \to \infty$, n goes to zero, being infinite in the opposite extreme ($T = 0$). Note, however, since the enthalpy is $H = (\rho + p)V$, the chemical potential is zero. The same happens to the specific heats.

Let us now consider the vacuum spectrum. From the above results we can say that the energy spectrum $\rho_T(\nu)$ must satisfy two thermodynamic constraints:

$$\rho = \int_0^\infty \rho_T(\nu)\, d\nu = \text{const} \tag{16}$$

and

$$n = \int_0^\infty \frac{\rho_T(\nu)}{h\nu}\, d\nu = \frac{\text{const}}{T} \tag{17}$$

In order to show how these constraints may be useful to determine the general form of the spectrum, we assume that it is described by a "Wien-type" law

$$\rho_T(\nu) = \nu^\beta \phi(\nu^\beta T^\lambda) \tag{18}$$

where ϕ is an arbitrary function of its argument and the powers β and λ will be fixed by the constraints (16) and (17). By defining a new variable $u = \nu^\beta T^\lambda$, it is straightforward to establish from (16) and (17) that

$$\rho = \frac{1}{T^{\lambda(1+\beta)/\beta}} \int_0^\infty f(u)\, du \tag{19}$$

$$n = \frac{1}{T^\lambda} \int_0^\infty g(u)\, du \tag{20}$$

where $f(u)$ and $g(u)$ are functions related to $\phi(u)$. Note that in the case of photons, $\rho \propto T^4$ and $n \propto T^3$, one obtains $\beta = -\lambda = 3$, recovering both results.[5] For the vacuum state, comparing equations (16) and (17) with (19)

[5]Three different methods, among them the one considered by Boltzmann (Carnot cycle), can be used to deduce that for a γ law the energy density is given by $\rho = \eta T^{\gamma/(\gamma-1)}$, where η is a γ-dependent constant (Lima and Santos, 1995).

and (20), respectively, we find $\lambda = 1$ and $\beta = -1$. Therefore, instead of the result $\rho_T(\nu) = \text{const} \cdot \nu^3$ claimed by the proponents of SED, we have found that the unique "Wien-type" spectrum for the vacuum state compatible with the thermodynamic constraints is given by

$$\rho_T(\nu) = \nu^{-1}\phi\left(\frac{T}{\nu}\right) \tag{21}$$

where ϕ is an arbitrary function of its argument.

It should be noticed that even in the limit $T \to 0$, $\rho_T(\nu)$ scales with ν^{-1} instead of ν^3, as usually inferred from the blackbody radiation spectrum (Marshall, 1963; Boyer, 1969, 1980; see also Sciama, 1991). The mistake of such an inferences lies in the fact that its results are really arguments in favor of a zero-point spectrum satisfying the equation of state $p = \frac{1}{3}\rho$. Indeed, since the vacuum energy density is not only constant but also Lorentz invariant, we have shown that the existence of a temperature-dependent spectrum for the vacuum state is not forbidden by the relativity principle, as long as the vacuum state is described by the equation of state $p = -\rho$.

The above results may also be interesting for early-universe physics, mainly for the so-called inflationary models. In fact, the essential feature of such models is the appearance of an accelerated expansion of the universe driven by the vacuum stress arising, for instance, from a scalar field with a global minimum in its effective potential (Linde, 1984) or some types of phase transitions (Sato, 1981; Guth, 1982; Linde, 1982). The inflaton field driving inflation behaves, under certain conditions, like a perfect fluid with $p = (\gamma - 1)\rho$, where $\gamma < 1$. However, as far as we know, its thermodynamic behavior has been neglected. In this regard, since the formalism is manifestly covariant, we can apply equation (13) for a Friedmann–Robertson–Walker (FRW) metric ($V \propto R^3$) we obtain

$$T = T_*\left(\frac{R_*}{R}\right)^{3(\gamma-1)} \tag{22}$$

where R is the universal scale function and $T_* = T(R_*)$ is the temperature of the universe stage specified by R_*. For $\gamma = 4/3$ one finds $T \propto R^{-1}$ as usual for a radiation-dominated phase. Important results are obtained for power-law inflation ($0 < \gamma < 1$, $T \propto R^{3(1-\gamma)}$) and exponential inflation ($\gamma = 0$, vacuum, $T \propto R^3$). It is also interesting that the treatment of the vacuum as a fluid interacting with the other matter fields (such as a multifluid model) is in line with recent attempts to solve the so-called cosmological constant problem (Weinberg, 1989). According to such an approach, the cosmological effective vacuum energy density relaxed to the present small value due to the universal expansion (Peebles, 1984; Chen and Wu, 1990; Carvalho *et al.*, 1992).

Concluding, some thermodynamic properties of the quantum vacuum including the general form of its spectrum have been established. Of course, as for blackbody radiation, the specific form of the arbitrary function contained in it must be established by statistical considerations. We also argued that the unsettled situation arising from the overall existence of the vacuum and its consequences in the interface uniting QFT, general relativity, and cosmology may be circumvented by a more comprehensive picture of the vacuum state itself. In this regard, we hope that the thermodynamic approach outlined here may be useful to point the way for a more fundamental description of the vacuum.

A more detailed account of this work, including thermomechanical properties and a rigorous derivation of "Wien's law" for the vacuum state, will be published elsewhere.

ACKNOWLEDGMENTS

We thank A. M. Mariz, B. Mashhoon, and A. Dolgov for helpful discussions. We are also grateful to the Brazilian research agency CNPq for partial financial support.

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